Development of Extreme Rainfall Based Intensity-Duration-Frequency Curves for Dhaka City in Bangladesh

Imran Hossain Newton, Abir Biswas, Md Mohiuddin Sakib, Shamrita Zaman, Naw Safrin Sattar, Rabeya Akter

Abstract— The continuous propagation of greenhouse gases in the hydrological cycle due to climate change is causing variations in intensity, duration and frequency of rainfall events in the third world countries. Due to the variation of rainfall intensity urban areas especially Dhaka city of Bangladesh often become water logged due to inadequate and poor drainage system. Therefore, Intensity Duration Frequency (IDF) curves for extreme rainfall events are effective for drainage design. The IDF curves were developed using daily rainfall data for different return periods i.e. (2, 5, 10, 25, 50 and 100 years) using the Extreme Value Type 1 (Gumbel) and Log Pearson Type III (LPT III) distribution considering rainfall intensity values of 24, 48, 72, 96,120, 144 and 168 hours respectively. The results obtained from LPT III distribution are slightly higher than the results obtained using the Gumbel distribution. Three different Goodness-of-fit (GOF) test were used to determine the best fit probability distribution. Result shows that the Gumbel distribution is best for the prediction of rainfall intensity for Dhaka city.

Index Terms— Extreme Value Type 1 (Gumbel), Goodness-of-fit (GOF), Intensity-Duration-Frequency(IDF), Log Pearson Type III, Rainfall intensity.

1 Introduction

THE initial stages of the drainage system design process involves the development of the rainfall Intensity-Duration-Frequency (IDF) curves [1]. The depth of rainfall collected from the intensity duration frequency curve is used for planning, designing and operation of water resource related projects [2]. The construction of such relationships had begun as early as 1932 [3].

A variety of relationships have been developed by other nations since then. However, the accurate construction of such relationships has been unsuccessful in many developing countries [2]. As a result of inadequate drainage facility, Dhaka has experienced a severe waterlogging and flooding problem during the monsoon period. In 2004, Dhaka city experienced its heaviest ever recorded rainfall (341 mm) and its demoralizing impact stopped the city life.

In order to solve this problem, new drainage design is required, which necessitates the attainment and analysis of rainfall data for different durations. Therefore, the objective of this study to develop intensity-Duration-Frequency (IDF) curves for Dhaka city for extreme rainfall events. In the present study, annual maximum rainfall series is considered for carrying out the Rainfall Frequency Analysis (RFA).

Rainfall in an area can be characterized if the intensity, duration, and frequency of the various storms occurring at that place are known [4]. The frequency data for rainfall of varying durations can be represented by IDF curves, which give a plot of rainfall intensity versus rainfall duration. In later studies, many people tried to relate IDF relationship to the brief meteorological conditions in the area of hydrometric stations [5][6]. IDF curve for North-East region of Bangladesh was developed and observations reveal that the rainfall data in this region follow Gumbel's Extreme Value Type 1 Distribution [7]. But less studies have been found regarding development of IDF curve for the urban cities. In this study, the IDF curves were developed for Dhaka city (latitude 23°42′37″N, longitude 90°24′26″ E) (Fig. 1) for 2, 5, 10, 25, 50 and 100 years return periods using the Extreme Value Type 1 (Gumbel) and Log Pearson Type III (LPT III) distribution for rainfall intensity values for durations of 24, 48, 72, 96,1 20, 144 and 168 hours. The chi-square, Kolmogorov-Smirnov test, Anderson-Darling goodness-of-fit tests were used to determine the best fit probability distribution.

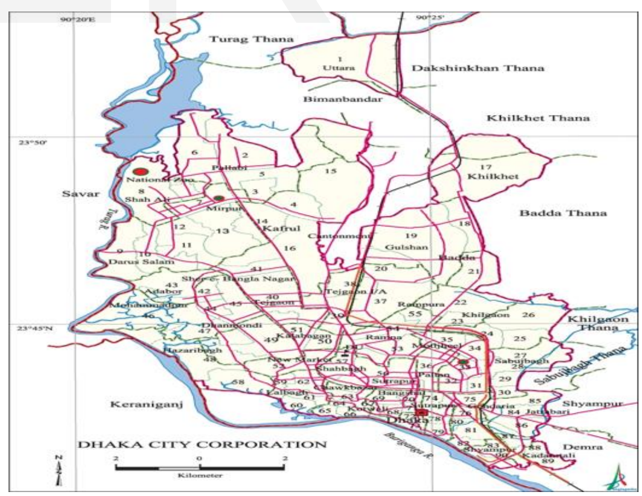

Fig.1 Study Area

2 Metholodology

In this study, the rainfall data set of 58 (1953-2010) years are obtained from the Bangladesh Meteorological Department (BMD) for Dhaka city. The data was analyzed using statistical methods of least squares using Microsoft Excel programming. The rainfall values for periods of 24, 48, 72, 96, 120, 144 and 168 hours were extracted from the analysis. Then, the IDF curves were developed for 2, 5, 10, 25, 50 and 100 years return

International Journal of Scientific & Engineering Research, Volume 8, Issue 1, June-2017
ISSN 2229-5518

period using the Extreme Value Type 1 (Gumbel) and Log Pearson Type III distribution. Goodness-of-fit (GOF) tests were used to determine the best fit probability distribution. It also helps to rank the fitted distributions according to quality of fit over the raw data. Most used Goodness-of-fit tests include Chi Squared Tests, Kolmogorov-Smirnov Test, Anderson-Darling Test, that we have used to determine the best fit in this study.

2.1 Gumbel Theory of Distribution

The Gumbel distribution calculates 2, 5, 10, 25, 50 and 100 years return periods for each duration period and requires few calculations. With a specified return period T (in year), the frequency precipitation P_T (in mm) is given by the following equation:

$$P_T = P_{avg} + KS \tag{1}$$

Where K denotes as Gumbel frequency factor and given in (2) and P_{avg} is the average of the maximum rainfall corresponding to a specific duration expressed in (3).

$$K = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left[\frac{T}{T-1}\right]\right]\right\} \tag{2}$$

$$\mathrm{P_{avg}} = \frac{1}{\mathrm{n}}\sum_{i=1}^{\mathrm{n}} \mathrm{P_i} \tag{3}$$

Where P_i is the individual extreme value of rainfall and n is the number of events or years of record. The standard deviation is calculated by (4) computed using the following relation:

$$\mathrm{S} = \sqrt{\left[\frac{1}{\mathrm{n-1}}\sum_{i=1}^{n}\left(P_i - P_{avg}\right)^2\right]} \tag{4}$$

Where S is the standard deviation of P data. K is a function of return period, which say as frequency factor, when multiplied by the standard deviation gives the departure of a chosen return period rainfall from the average. Then, the rainfall intensity I_T (in mm/hr) for return period T is obtained from (5).

$$I_T = \frac{P_T}{T_d} \tag{5}$$

T_d Denotes the duration in hours.

The annual maximum series, which consists of highest values recorded each year, is used as a reference to describe the frequency of the rainfall. The peak-over threshold concept, which only uses rainfall amounts beyond a certain threshold for various durations is also another data format for rainfall frequency studies. The popularity of annual maximum series method in practice, is mainly due to its modest structure [8].

2.2 Log Pearson Type III

The rainfall intensity is also calculated using the LPT III probability model for the different rainfall durations and return periods to generate the IDF curves for any station. The first step of LPT III distribution consist of transforming the observed values by taking their logarithms. The mean and standard deviation of the logarithmically transformed data are also derived. The frequency precipitation is attained using LPT III method, in the same manner as Gumbel distribution. The simplified expression for this distribution is given as follows:

$$P^* = \log(\mathrm{P_i}) \tag{6}$$

$$P^*_T = P^*_{avg} + K_T S^* \tag{7}$$

$$P^*_{avg} = \frac{1}{n}\sum_{i=1}^{n} P^* \tag{8}$$

$$\mathrm{S}^* = \left[\frac{1}{\mathrm{n-1}}\sum_{i=1}^{\mathrm{n}}\left(P^* - P^*_{avg}\right)^2\right]^{\frac{1}{2}} \tag{9}$$

Where P^*_T, P^*_{avg} and S^* are expected rainfall peak value for a specific frequency, average of maximum precipitation corresponding to a specific duration and standard deviation of P^* value respectively. K_T denotes as Pearson frequency factor, which depends on return period (T) and skewness coefficient (C_s) .Which is required to compute the frequency factor for this distribution. The skewness coefficient is calculated using (10) [9] [10].

$$Cs = \frac{n\sum_{i=1}^{n}\left(X^*_i - \overline{X^*}\right)^3}{(n-1)(n-2)\left(S^*\right)^3} \tag{10}$$

Where, K_T values can be acquired from tables in many hydrology books [9]. Through knowing the C_s and the recurrence interval, the frequency factor, K_T for the LPT III Distribution can be extracted. The antilog of P^*_T will provide the estimated extreme value for the given return period and the rainfall intensity I_T (in mm/hr).

2.3 Chi-Square Test (C-S)

The objective of this test is to determine how good is a fit between the observed frequency of occurrence in a sample and the expected frequencies obtained from the hypothesized distributions. The chi-square quantity, which represents the goodness of fit between observed and expected frequency, is expresses as follows:

$$X^2 = \frac{\sum_{i=1}^{k}\left(O_i - E_i\right)^2}{E_i} \tag{11}$$

Where X^2 is a random variable whose sampling distribution is approximated very closely by the chi-square distribution. The symbols O_i and E_i represent the observed and expected frequencies, respectively, for the i-thclass interval in the histogram. The symbol K represents the number of class intervals. The smaller the differences between observed and expected frequency, the better the fit. A better fit leads to the acceptance of the null hypothesis, whereas a poor fit leads to a rejection.

International Journal of Scientific & Engineering Research, Volume 8, Issue 1, June-2017
ISSN 2229-5518

For a particular level of significance 'a', the critical value is readily available in the chi-square tables, where a value greater than X^2 represents the critical region [11][12].

2.4 The Kolmogorov-Smirnov Test (K-S)

The Kolmogorov's Statistic is defined as

$$d = Max\left|F(x) - E(x)\right| \qquad (12)$$

Where, $F(x)$ and $E(x)$ are the theoretical and empirical functions evaluated at x, respectively.
These two functions are evaluated at x_i are defined as

$$F(x_i) = P(X \leq x_i) \qquad (13)$$

And

$$E(Xi) = \frac{\# of X's \leq xi}{n} = \frac{i}{n} \qquad (14)$$

$$i = 1,2,3.....n$$

If the observed maximum departure d is small, then the assumed $F(x)$ may be reasonable as that distribution that generated the data. But if this d is "large" then it is unlikely that $F(x)$ is the underlying data distribution [13].

2.5 Anderson-Darling Test (A-D)

The Anderson-Darling test [14] is used to test if a sample of data came from a population with a specific distribution. It is a modification of the Kolmogorov-Smirnov (K-S) test and gives more weight to the tails than does the K-S test.
The Anderson-Darling statistics A^2 is defined as

$$A^2 = -n - \frac{1}{n}\sum_{i-1}^{n}(2i-1).\left[\ln F(x_i) + \log\left(1 - F\left(X_{n-i+1}\right)\right)\right] \qquad (15)$$

3 Result and Discussion

The objective of this study was to develop IDF curves of varying durations and return periods for Dhaka city. Whenever any engineering project involves designing drainage structures, the IDF curves become necessary, which allows them to design sufficiently safe and economical flood control measures. The rainfall estimates and intensities measures in mm and mm/hr, respectively, for the various return periods and durations were analyzed using Gumbel and LPT III distributions. The results are listed in Table 1 and Table 2.

Table 1 COMPUTED FREQUENCY PRECIPITATION (P_T) VALUES AND RAINFALL INTENSITY (I_T) FOR DIFFERENT DURATIONS AND RETURN PERIODS USING GUMBEL DISTRIBUTION FOR DHAKA CITY

Duration (hour)	T (year)	K	P_T (mm)	I_T (mm/hr)	P_{avg} mm	S
24	2	-0.16	132.44	5.52	141.87	57.42
	5	0.72	183.18	7.63		
	10	1.30	216.78	9.03		
	25	2.04	259.23	10.80		
	50	2.59	290.72	12.11		
	100	3.14	321.98	13.42		
48	2	-0.16	180.39	3.76	193.04	77.00
	5	0.72	248.44	5.18		
	10	1.30	293.49	6.11		
	25	2.04	350.41	7.30		
	50	2.59	392.64	8.18		
	100	3.14	434.55	9.05		
72	2	-0.16	209.77	2.91	223.48	83.42
	5	0.72	283.49	3.94		
	10	1.30	332.30	4.62		
	25	2.04	393.98	5.47		
	50	2.59	439.73	6.11		
	100	3.14	485.14	6.74		
96	2	-0.16	232.16	2.42	247.22	91.66
	5	0.72	313.16	3.26		
	10	1.30	366.79	3.82		
	25	2.04	434.55	4.53		
	50	2.59	484.81	5.05		
	100	3.14	534.71	5.57		
120	2	-0.16	246.45	2.05	262.13	95.46
	5	0.72	330.81	2.76		
	10	1.30	386.66	3.22		
	25	2.04	457.23	3.81		
	50	2.59	509.58	4.25		
	100	3.14	561.55	4.68		
144	2	-0.16	262.48	1.82	278.78	99.20
	5	0.72	350.15	2.43		
	10	1.30	408.19	2.83		
	25	2.04	481.53	3.34		
	50	2.59	535.94	3.72		
	100	3.14	589.94	4.10		
168	2	-0.16	275.97	1.64	292.83	102.64
	5	0.72	366.68	2.18		
	10	1.30	426.73	2.54		
	25	2.04	502.61	2.99		
	50	2.59	558.91	3.33		
	100	3.14	614.78	3.66		

TABLE 2 COMPUTED FREQUENCY PRECIPITATION (P^*_T) VALUES AND RAINFALL INTENSITY (I_T) FOR DIFFERENT DURATIONS AND RETURN PERIODS USING LPT III FOR DHAKA CITY

Duration (hour)	T (year)	K	P^*_T (mm)	I_T (mm/hr)	P^*_{avg} mm	S^*
24	2	-0.08	128.59	5.36	4.89	0.36
	5	0.80	177.16	7.38		
	10	1.32	213.69	8.90		
	25	1.92	265.21	11.05		
	50	2.33	307.47	12.81		
	100	2.71	352.78	14.70		
48	2	-0.08	175.49	3.66	5.20	0.36
	5	0.80	241.11	5.02		
	10	1.32	290.08	6.04		
	25	1.92	358.70	7.47		

International Journal of Scientific & Engineering Research, Volume 8, Issue 1, June-2017
ISSN 2229-5518

	50	2.33	414.64	8.64		
	100	2.71	474.33	9.88		
72	2	-0.08	207.59	2.88	5.35	0.36
	5	0.80	281.63	3.91		
	10	1.32	332.89	4.62		
	25	1.92	400.30	5.56		
	50	2.33	452.11	6.28		
	100	2.71	504.85	7.01		
96	2	-0.08	229.15	2.39	5.45	0.35
	5	0.80	310.47	3.23		
	10	1.32	367.43	3.83		
	25	1.92	443.09	4.62		
	50	2.33	501.80	5.23		
	100	2.71	562.01	5.85		
120	2	-0.08	243.39	2.03	5.51	0.35
	5	0.80	328.06	2.73		
	10	1.32	387.29	3.23		
	25	1.92	465.92	3.88		
	50	2.33	526.92	4.39		
	100	2.71	589.47	4.91		
144	2	-0.08	259.62	1.80	5.57	0.34
	5	0.80	347.91	2.42		
	10	1.32	409.30	2.84		
	25	1.92	490.41	3.41		
	50	2.33	553.06	3.84		
	100	2.71	617.08	4.29		
168	2	-0.08	273.38	1.63	5.62	0.33
	5	0.80	364.96	2.17		
	10	1.32	428.24	2.55		
	25	1.92	511.40	3.04		
	50	2.33	575.33	3.42		
	100	2.71	640.42	3.81		

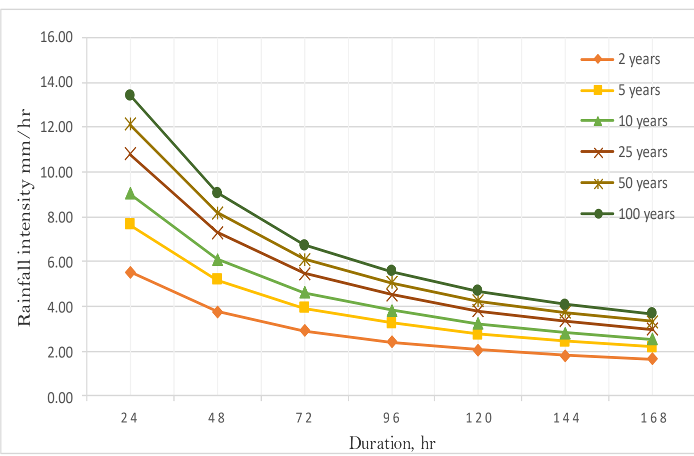

Fig. 2 IDF curve by Gumbel method for Dhaka city

According to the IDF curves, rainfall estimates are found to be greater with an increase in the return period and the rainfall intensities decline with rainfall duration in all return periods. The results obtained from the two different methods are relatively consistent. Fig. 2 and Fig. 3 show results of the IDF curves obtained by Gumbel and LPT III distribution methods for Dhaka city respectively. It was found that there were minor differences between the results obtained from the two methods, where LPT III distribution are slightly higher than the results obtained using the Gumbel distribution.

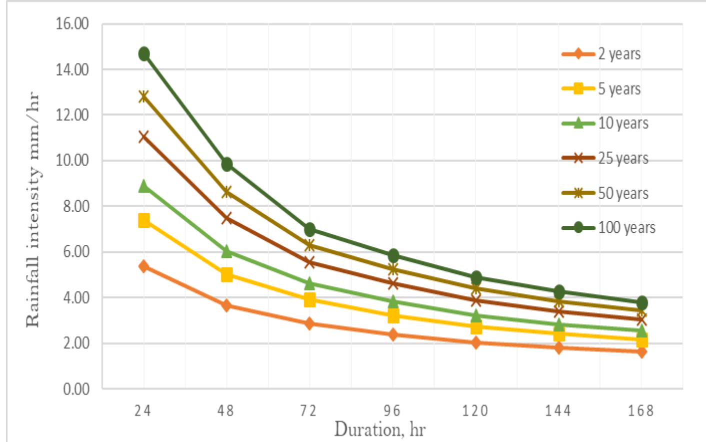

Fig. 3 IDF curves by LPT III method for Dhaka city

The best fitted distribution has been determined using GOF tests. According to the GOF tests, the best distribution for annual maximum rainfall was selected based on the least deviations produced. Results of GOF tests for annual maximum rainfall in different durations are shown in Table 3.

TABLE 3 GOF TESTS RESULTS FOR ANNUAL MAXIMUM RAINFALL IN DIFFERENT DURATIONS FOR DHAKA CITY

Duration (hour)	Gumbel			LPTIII			Selected distribution
	C-S	K-S	A-D	C-S	K-S	A-D	
24	6.3996	0.1023	0.6813	4.6713	0.0888	0.4521	LPTIII
48	0.7949	0.0641	0.4599	2.3171	0.0578	0.1718	LPTIII
72	1.1530	0.0625	0.2654	1.9732	0.0681	0.2751	Gumbel
96	0.9802	0.0699	0.3158	2.7895	0.0732	0.3408	Gumbel
120	2.9791	0.0626	0.2793	3.8312	0.0695	0.2831	Gumbel
144	4.0099	0.0663	0.3620	4.3911	0.0729	0.3700	Gumbel
168	1.9072	0.0755	0.3825	2.7054	0.0780	0.4016	Gumbel

The goodness-of-fit tests reveal that Log-Pearson Type III is more suitable for 24-hour duration rainfall, whereas the Gumbel Distribution provides a better fit for 72, 96, 120, 144 and 168-hour rainfall in all three tests. However, in the case of 48-hour rainfall, the K-S test and A-D test prove that Log-Pearson Type III distribution is a better fit, but the chi-square test says otherwise.

In most cases, the values for the K-S test and A-D test for the two distributions are very similar to each other, which means that either of the distribution could be a better fit. However, in the cases, where all three tests agree with each other, a distribution can be said to be of a better fit. With given rank for each GOF test it seems that Gumbel distribution is best for the prediction of rainfall intensities for Dhaka city.

International Journal of Scientific & Engineering Research, Volume 8, Issue 1, June-2017
ISSN 2229-5518

4 Conclusions

Intensity-duration-frequency data is a necessity for hydrologists and engineers involved in the planning and design phases of water resources projects. Historical rainfall records obtained from Bangladesh Meteorological Department (BMD) were used to generate the IDF curves for Dhaka region. The maximum intensity of any rainstorm over any return period in Dhaka can be obtained from the IDF curve. For both Gumbel and LPT III distribution methods, IDF curves follow the same pattern. It is clear that with the increased duration (hr), rainfall intensity decreases simultaneously. on the other hand, with the increase of return periods (year), rainfall intensity (mm/hr) increases gradually. The expected rainstorm intensity can be used to evaluate the value of discharge and hence the design of hydraulic structures also. This IDF curves are also highly recommended for the prediction of rainfall intensities in this region.

Acknowledgment

The authors are thankful to the Bangladesh Meteorological Department (BMD) for providing the rainfall data of Dhaka city, Bangladesh.

References

[1] Chawathe, SD, Shinde, UR, Fadnavis, SS, & Goel, VV. (1977). Rainfall Analysis for the Design of Storm Sewers in Bombay. IE Journal (EN), 58, 14-20.

[2] Koutsoyiannis, Demetris, Kozonis, Demosthenes, & Manetas, Alexandros. (1998). A mathematical framework for studying rainfall intensity-duration- frequency relationships. Journal of Hydrology, 206(1), 118-135.

[3] Bernard, Merrill M. (1932). Formulas for rainfall intensities of long duration. Transactions of the American Society of Civil Engineers, 96(1), 592-606.

[4] Bougadis, John, & Adamowski, Kaz. (2006). Scaling model of a rainfall intensity-duration-frequency relationship. Hydrological Processes, 20(17), 3747-3757.

[5] Dupont, BS, & Allen, DL. (2006). Establishment of Intensity–Duration–Frequency Curves for Precipitation in the Monsoon Area of Vietnam. Kentucky Transportation Center, College of Engineer, University of Kentucky in corporation with US Department of Transportation.

[6] Matin, MA, & Ahmed, SMU. (1984). Rainfall Intensity Duration Frequency Relationship for the NE Region of Bangladesh. Journal of Water Resource Research, 5(1), 4.

[7] Chowdhury, RK, Alam, MJB, Das, P, & Alam, MA. (2007). Short duration rainfall estimation of Sylhet: IMD and USWB method. JOURNAL-INDIAN WATERWORKS ASSOCIATION, 39(4), 285.

[8] Burke, CB, & Burke, TT. (2008). Storm Drainage Manual. Indiana LTAP.

[9] Chow, V.T. (1988). Handbook of Applied Hydrology: McGraw-Hill Book.

[10] Rashid, MM, Faruque, SB, & Alam, JB. (2012). Modelling of short duration rainfall intensity duration frequency (SDRIDF) equation for Sylhet City in Bangladesh. ARPN Journal of Science and Technology, 2(2), 92-95.

[11] Al-Shaikh, A.A. (1985). Rainfall frequency studies for Saudi Arabia, M.S Thesis, Civil Engineering Department. King Saud University Riaydh (K.S.A).

[12] Oyebande, Lekan. (1982). Deriving rainfall intensity-duration- frequency relationships and estimates for regions with inadequate data. Hydrological Sciences Journal, 27(3), 353-367.

[13] Wang, Hsiao-Mei. (2009). Comparison of the Goodness-of- Fit Tests: the Pearson Chi-square and Kolmogorov-Smirnov Tests. Metering management journal', Volume (06).

[14] Stephens, Michael A. (1974). EDF statistics for goodness of fit and some comparisons. Journal of the American statistical Association, 69(347), 730-737.

- *Imran Hossain Newton is currently a Graduate Student in Institute of Water and Flood Management, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, e-mail: newton.buet.iwfm@gmail.com*
- *Abir Biswas is currently a Graduate Student in Institute of Water and Flood Management, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, e-mail: abiswas3128@gmail.com*
- *Md Mohiuddin Sakib is currently a Research Associate in Institute of Water and Flood Management, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, e-mail: sakib.mohiuddin06@gmail.com*
- *Shamrita Zaman is currently a Research Assistant in Institute of Water and Flood Management, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, e-mail: chowdhuryshamrita@yahoo.com*
- *Naw Safrin Sattar is currently a Graduate Student in Computer Science Department, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, e-mail: 1005099.nss@ugrad.cse.buet.ac.bd*
- *Rabeya Akter is currently a Research Fellow in Institute of Water and Flood Management, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, e-mail: rabeya343@gmail.com*